# Anomalous Phase observed in MT response computed over an Elongated Prism Body: A Synthetic 3D MT Forward Modelling 

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#### Abstract

SUMMARY In the Magnetotelluric (MT) method, orthogonal components of the electric field and magnetic field are measured and transformed into the transfer functions such as impedance tensor, apparent resistivity and phase etc. Usually, phases for the off-diagonal components of the impedance tensor lie in the first $\left(0^{\circ}<\phi<90^{\circ}\right)$ and third quadrant ( $-180^{\circ}<\phi<-90^{\circ}$ ) for the XY and YX components, respectively. Sometimes, phases may lie out of these quadrants. Such phases are termed Anomalous phases (AP). The AP acts as a barrier to the proper interpretation of the MT data. Simple 1D or 2D structures do not cause AP. Previously, complex 3D models are used to demonstrate and discuss the AP. This study shows that a simple 3D model can generate AP. A conductive prism orientated $\mathrm{N} 60^{\circ} \mathrm{W}$ is embedded in half space at a shallow depth. The impedance tensor is computed at a regular grid. We observed AP near the two edges of the prism in the simulated data. The electric field vector rotates more than $90^{\circ}$ at these places in a period greater than 10 s . The simple structure is able to produce the AP due to the current channeling toward the conductor and shows reversal of the electric field. The AP depends on the length/width ratio and orientation of the prism and the conductivity contrast between the prism and the background. Further, a discontinuity was introduced at the center of the prism. Using the computed forward responses, AP is also observed around the discontinuity region. This study can be used as a step towards the generalization of AP.


Keywords: Magnetotelluric, Anomalous phase, 3D forward modelling, elongated prism body.

## INTRODUCTION

Magnetotelluric method is used for investigating the subsurface electrical resistivity structure by simultaneously measuring orthogonal components of electric, $\mathbf{E}$, and magnetic, $\mathbf{H}$, field on the earth's surface. The impedance tensor $(\mathbf{Z})$ is linearly related as
$\left[\begin{array}{l}E_{x} \\ E_{y}\end{array}\right]=\left[\begin{array}{ll}Z_{x x} & Z_{x y} \\ Z_{y x} & Z_{y y}\end{array}\right]\left[\begin{array}{l}H_{x} \\ H_{y}\end{array}\right]$
Where, $E_{x}, E_{y}$ are the two orthogonal components of Electric field and $H_{x}, H_{y}$ are the two orthogonal components of magnetic field. $Z_{x x}, Z_{y y}$ are the diagonal and $Z_{x y}, Z_{y x}$ are the off-diagonal components of impedance tensor. The phases for the off-diagonal components, $Z_{x y}$ and $Z_{y x}$ components, normally lie in the first ( $0^{\circ}<\phi<90^{\circ}$ ) and third quadrant ( $-180^{\circ}<\phi<-90^{\circ}$ ), respectively. In some cases, the phases lie out of these quadrants is called the Anomalous phase (AP). Previously, the cause of AP is identified due (1) the presence of electrical anisotropy (Heise and Pous 2003), (2) Galvanic distortion (Chouteau and Tournerie 2000; Lilley and Weaver 2010), (3) 2D structure with large resistivity contrasts (Selway et al. 2012) and (4) 3D conductivity bodies (Lezaeta and Haak 2003;

Ichihara and Mogi 2009; Ichihara et al. 2013). In this study, we developed a 3D conductivity model that also resulted in generating AP. This is a 3D model contains a single horizontal conductive rectangular prism in a homogeneous resistive environment. Numerous experiments were performed by varying the resistivity, size and orientation of the prism to study their effect in generating AP. It was also found that if a discontinuity is introduced within the body such as the discontinuous region can be traced out using the Anomalous phase.

## Synthetic 3D Model

A 3D model with a single horizontal and conductive rectangular prism is presented in Figure 1. The prism is 100 km long with width and thickness of 10 km and is buried the top of prism at a depth of 1 km . The prism is oriented at the azimuth of $\mathrm{N} 60^{\circ} \mathrm{W}$. The resistivity of the prism and halfspace are taken $1 \Omega \mathrm{~m}$ and $1000 \Omega \mathrm{~m}$, respectively. The model is developed in Comsol Multiphysics software (Li and Butler 2021). In the software the forward simulations are based on finite element method, thus making it flexible for simulating complex geometries.


Figure 1: The Conductive rectangular prism model. The prism is oriented $\mathrm{N} 60^{\circ} \mathrm{W}$ placed at shallow depth of 1 km . (a) Map view at depth of 1 km . (b) 3D view of the model.

The synthetic data for 25 periods between $10^{3} \mathrm{~Hz}$ to $10^{3} \mathrm{~s}$ is computed at the edges of the elements located at the surface ( $z=0 \mathrm{~km}$ ). The simulated impedance phase value for $Z_{x y}$ and $Z_{y x}$ is plotted in the color scale in Figure 2a and 2c, respectively. The impedance phase for the $Z_{y x}$ component is taken to the first quadrant by adding $180^{\circ}$. The normalized electric field vector is also shown in the same plots (left panels). The AP can be observed at the two corners A and B of the prism for the $Z_{x y}$ component and two edges C and D of the prism for the $Z_{y x}$ component. At the AP region, there is a reversal in the E-field which means that the E-field is rotated by more than $90^{\circ}$ angle from the direction of the regional E-field vector. The apparent resistivity and phase data are plotted for sites (A and $B$ ) in Figure $2 b$ and sites ( $C$ and $D$ ) in Figure 2d. The phase is out of the quadrant for the period range 10 - 1000 s for all four sites. At the period of 1000 s , the phase for both $Z_{x y}$ and $Z_{y x}$ components crossed two quadrants from their principal quadrants. The phase value at sites $A$ and $B$ are $153^{\circ}$ and $-163^{\circ}$ respectively and at sites $C$ and $D$ are $216^{\circ}$ and $217^{\circ}$, respectively in the period of 1000 s. The presented model shows the AP around half of the long-sides of the prism body for $Z_{y x}$ component. We will continue with variation in phase for the $Z_{y x}$ component for further experiment.


Figure 2: The Impedance phase is plotted in color along with the normalized E-field vector for (a) $Z_{x y}$ component and (c) $Z_{y x}$ component at 1000 s period. The apparent resistivity and phase data are plotted for off-diagonal impedance at the sites $A$ and $B$ in (b) and at sites $C$ and $D$ in (d).

The variation in the physical properties of the prism makes it more generalized to understand. Hence, we have incorporated variations in the prism like the orientation of the prism, width/length (W/L) ratio, dimensions scale and resistivity contrast.

## Rotation of The Prism

The prism is rotated at each $15^{\circ}$ angle from EW to NS, as shown in Figure 3(a-g). Near the prism, the regional direction of the E-field vector is rotated towards the prism because of its conductive properties. If the prism rotates $15^{\circ}$ clockwise, the E field have to be rotated more than $90^{\circ}$ around the half-length of the long-side of the prism and hence, generates the AP (Figure 3b). In the next three rotation ( $30^{\circ}, 45^{\circ}$ and $60^{\circ}$ ), the AP region shrinks towards the end, as seen in Figures 3(c-e). The AP disappears for the orientation angle of more than $60^{\circ}$. The results show that if the elongated conductive body is oriented at an angle from $15^{\circ}$ to $60^{\circ}$ from EW direction, the AP will be generated for the $Z_{y x}$ component at 1000 s period.


Figure 3: (a) The EW oriented rectangular prism is rotated at angle (b) $15^{\circ}$, (c) $30^{\circ}$, (d) $45^{\circ}$, (e) $60^{\circ}$, (f) $75^{\circ}$, (g) $90^{\circ}$ clockwise. The impedance phase and E-field vector are plotted for $Z_{y x}$ component at period 1000 s.

## Effect of Width/Length Ratio

In the next experiment, the width of the prism is varied ( $5,10,20$ and 50 km ) while the length is kept constant at 100 km , as shown in Figure 4(a-d). The AP is shown in the first three models. The fourth model has a few areas where the AP exists. The area of the AP declines as the W/L ratio increases. The anomalous phase is almost negligible in the fourth case, where the $W / L$ ratio is 1:2.


Figure 4: The width of the prism is (a) 5 km , (b) 10 km , (c) 20 km and (d) 50 km whereas the length is 100 km . The impedance phase and E-field vector are plotted for $Z_{y x}$ component at 1000 s .

## Conductivity Discontinuity in the Prism

In this experiment, the rectangular prism is divided into two parts by introducing a discontinuity between the prisms (Figure 5b). The body acts as the two separate rectangular prisms and shows the AP around the discontinuous region. In the field geological conditions, if a conductive dyke is oriented $\mathrm{N} 45^{\circ} \mathrm{W}$ and is discontinuous due to tectonic activities e.g., faulting, or fracturing, then the discontinuous region can be identified at the site showing the AP.


Figure 5: A conductive prism model with 5 km width is shown (a) without discontinuity and (b) with discontinuity in the middle. The phase for $Z_{y x}$ component and E-Field vector are plotted at period 1000 s.

## Conclusions

In this paper, we have introduced a 3D model responsible for generating an Anomalous Phase (AP). In the model, a rectangular conductive prism is embedded into a resistive halfspace. The prism is orientated $\mathrm{N} 60^{\circ} \mathrm{W}$. The AP is observed around half of the long-side of the prism. In this region, the Efield vector is rotated by more than $90^{\circ}$ angle from the regional direction. The AP is observed at different orientations $\left(15^{\circ}, 30^{\circ}, 45^{\circ}\right.$ and $60^{\circ}$ angle from the East) of the prisms in YX component of impedance. The effect of width to length (W/L) ratio reveals that the $A P$ is observed up to $W / L$ ratio 1:2. The AP is also generated when there is a discontinuity present in an elongated conductive body.

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