Electromagnetic Modeling Using Adaptive Grids - A Reflection on the Term Geometry

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SUMMARY

The goal of electromagnetic (EM) modeling is to find mathematical and physical models that represent ‘reality’ as well as possible and allow us to predict and understand the behavior of EM fields in matter-filled space. Inevitably, these models are always an abstract and incomplete representation of reality. They therefore contain sources of error and are subject to a limited range of validity. Particularly, the geometry of our physical models in the Earth sciences is extremely challenging due to its multi-scale nature.

On a small scale, we may end up in microscopic, possibly self-similar structures where non-inductive polarization effects are not covered anymore by our chosen mathematical model due to a variety of pore-scale and inner-surface electrochemical processes. Electric conductivity can become complex-valued, frequency dependent and non-linear. Moreover, texture or heterogeneity below a certain spatial scale can appear as anisotropy, leading to a matrix representation of electromagnetic parameters in the equations to be discretized. Much of this is subject to current petrophysical research, which gives us new insights into the fundamental properties of these parameters.

On a large scale, and this is actually the main focus of my talk, there are sources of error associated with the representation of macroscopic geometry and its discretization. Whereas geometry modeling started with homogeneous and layered halfspaces with only a handful of parameters in the first half of the 20th century, we were able to approximate 2D and 3D geometries in the 70s, 80s, and 90s using, in the end, millions of rectangular building blocks to construct geometric models in a brick-like and conformal fashion. This is simple and easy to handle but sometimes also inefficient, particularly when it comes to adopting to the geometric idiosyncrasies of our geo-reality. Staircase-like structures associated with rectangular Cartesian grids tend to introduce artifacts into the model response once the configuration of sources and receivers generates sensitivity towards the artificial geometric features. Therefore, more elegant ways of representing geometry evolved during the last 20 years incorporating unstructured and non-conforming grids or, recently, even meshless approaches.

I will focus on the numerical and computational side of modeling here and restrict myself to the classical scalar electromagnetic parameter set taking into account that computers will continue to expand our capabilities in the future. Since there are no closed solutions to Maxwell’s equations with spatially arbitrarily varying coefficients, discrete solutions inevitably imply approximation errors whose magnitude is generally difficult to assess. A priori and a posteriori error estimators have provided us with some means to quantify the accuracy of the computed solution or, at least, allow us to tailor the properties of the used discretization to a desired level. Moreover, we realize that the description of the geometry itself, i.e., before we start discretizing it, represents a major problem. We need tools to disassemble the computational domain into ‘water-tight’ subdomains with arbitrary shapes. Large-scale structures, like underground voids, boreholes, bathymetry, or the topography of the Earth’s surface can significantly influence the electromagnetic response within a specific frequency range or time interval. Ultimately, this means we have to deal with a multi-scale issue which requires an appropriate level of geometric detailedness and a flexible local refinement or coarsening to account for the true physical response once a reliable mathematical model has been chosen. It turns out that the sensitivity plays an important role to steer and direct the degree of resolution of meshes and geometry.

Due to brevity, the talk will not address the inverse problem and efforts to quantify uncertainty in a Bayesian sense. The talk much more tries to review some attempts to gain control over the error sources in the forward process, particularly with respect to error estimators defined within the theory of finite elements. I will briefly review the different types of finite elements used in our approaches to model vector electromagnetic fields and scalar potentials, and discuss the types of meshes underlying the discretization schemes with respect to their ability to represent arbitrary geometry.

Keywords: Finite elements, unstructured grids, error estimators, geometry modeling, level of detailedness, adaptivity